LSA MATHEMATICS
UNIVERSITY OF MICHIGAN

inquiry based learning michigan mathematics

Nina White

Individual and Whole-Class Textbooks/Journals in IBL Math Courses

Math 310 (Choice and Chance) & Math 431 (Euclidean Geometry for Future Secondary Teachers)

Math 310: Choice and Chance

"Every day the media showers us with news, analysis, and op-eds, which use and misuse numbers to arrive at various far-reaching conclusions. The objective of the course is to help students to acquire some basic mathematical skills to navigate in the sea of numbers. Often, this boils down to understanding a few fundamental, ancient, and deep concepts: randomness, fairness, coincidence, and bias."

Individual Textbook Description

In place of exams, you will compile your worksheet solutions into the form of a textbook. (15% "midterm", 30% "final")

- Midterm feedback: instructor randomly chooses two chapters (of 1-4) to grade.
- Final grade: students choose one chapter to be graded (of 5-9) and instructor randomly chooses one to grade.

Formatting 10 points
Completeness 20 points
Clarity 20 points
Content 50 pts
Creativity +3 pts

Content: This score will be broken down into
Introductory ideas and definitions (10pts)
Examples (15-20pts)
Proofs of theorems (15-10pts)
Exercises (10pts)

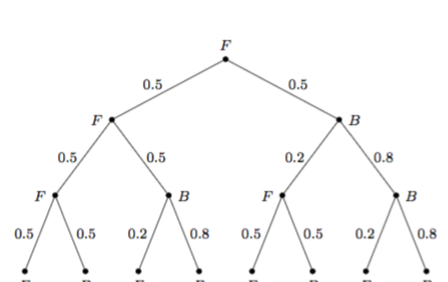
The introductory ideas and definitions, examples, and proofs of theorems come from our in-class work and discussions. For the ***Exercises*** *category, you must include 5 exercises at the end of each chapter to check the readers' understanding. The exercises should be relevant and accessible.*

Excerpts

9.1. 55

Example 9.1.3 You look out the window and see the goldfinch from the previous example on the bird feeder. What is the probability that it will be at the bird feeder in 3 minutes?

The following diagram shows the possible trajectories of the goldfinch in 3 minutes beginning at the bird feeder. The numbers next to each line represent the probability of each transition occuring.

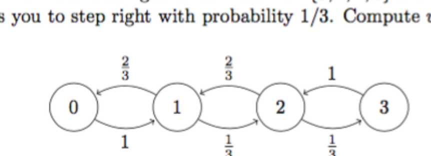


3	1	0	3	0	0
4	0	4	0	3	3

To find $v(4,\sigma)$ where $\sigma = 3b$, again we look at row 4. We found that there are 0 ways to end at 0, 4 ways to end at 1, 0 ways to end at 2, 3 ways to end at $3a$, and 3 ways to end at $3b$. Then since all trajectories are equally likely,

$$v(4,3b) = \begin{bmatrix} 0 \\ \frac{4}{10} \\ 0 \\ \frac{3}{10} \\ \frac{3}{10} \end{bmatrix}.$$

Example 8.1.7 You're doing a random walk on $\{0,1,2,3\}$ with an unfair coin, which causes you to step right with probability 1/3. Compute $v(4,0)$.

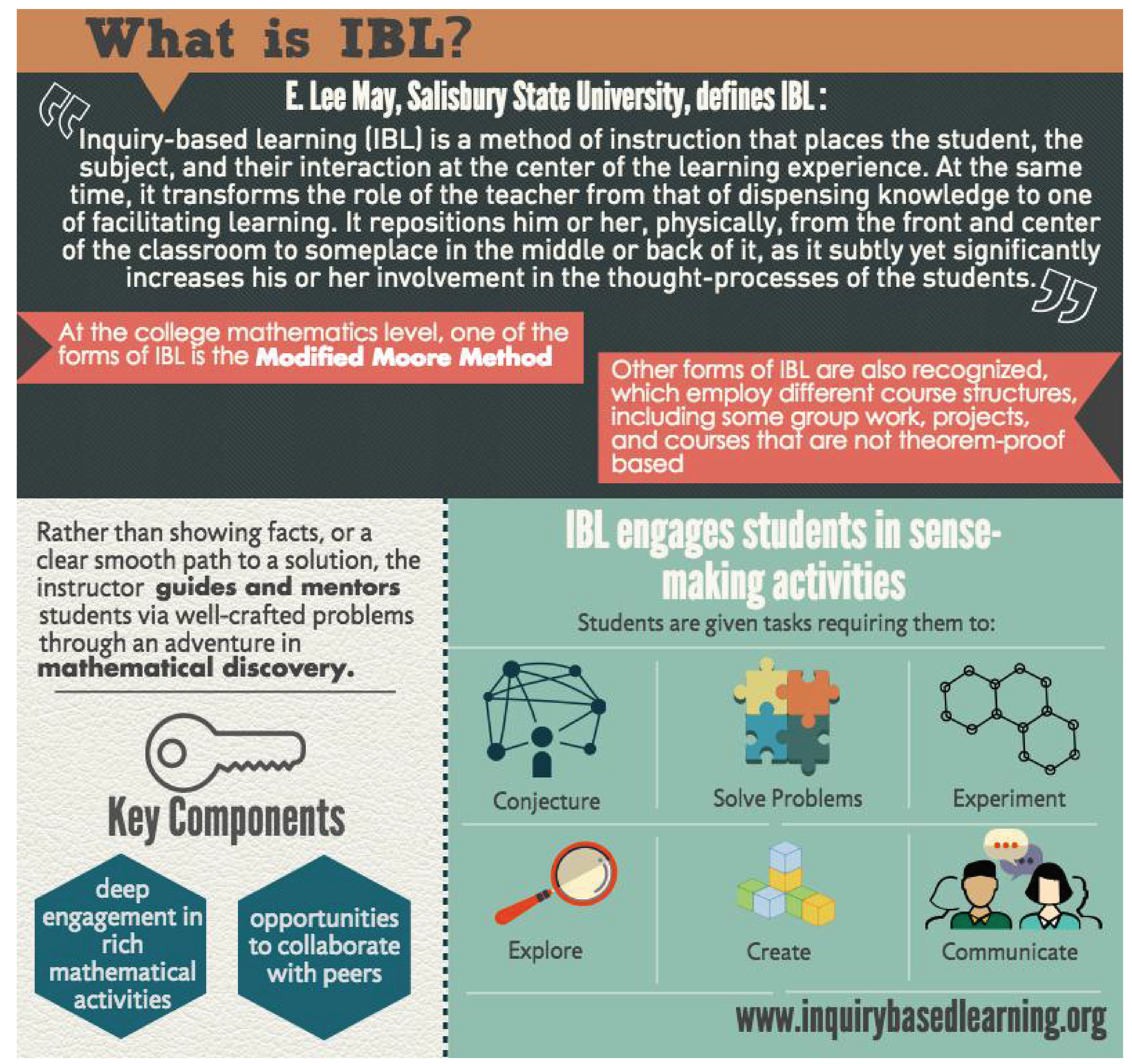


To compute $v(4,0)$ we need the probabilities of ending at each location after 4 steps given that we started at 0. For our first step, our only option is to step from 0 to 1, so this happens with probability 1.

Challenges and Changes

Too much time/work. **Change?** Class textbook or small groups. However, then students aren't accounatble for all material.

Communicating expectations.**Change?** Improved rubric. However, it's challenging to express expectations perfectly until confronted with weak examples.



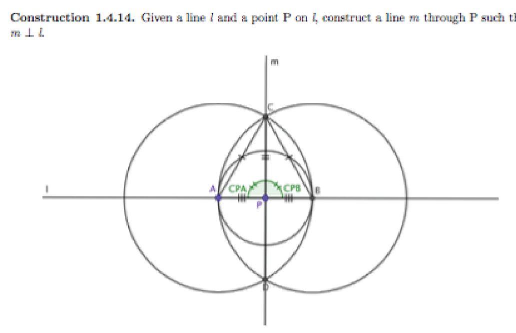
To affirm the value of student-driven, collaborative, mathematical problem-solving and communication in IBL courses, assessment must be reimagined; it must go beyond typical, timed, individual, math exams.

Excerpts

Construction 14: Create a line perpendicular to another line through a point on line

Nick Robbins

Construction 1.4.14. Given a line l and a point P on l, construct a line m through P such that $m \perp l$.



Proof. Begin with the line l through arbitrary points A and P. Create a circle $\odot PA$ centered at P with radius congruent to PA. Create a point B at the intersection of l and $\odot PA$ opposite of A. Thus $PA \cong PB$, because both A and B are on the circumference of $\odot PA$ Next, create a circle $\odot AB$ centered at A with radius congruent to line segment AB and a circle $\odot BA$ centered at B with radius congruent to BA.

At one intersection of $\odot AB$ and $\odot BA$, create a point C. We create line segments AC, BC, and CP from the corresponding points. We know that $AB \cong AC$ because B and C are on the circumference of $\odot AB$. Similarly, $BA \cong BC$. Since $AB \cong BA$, $AC \cong BC$ by transitivity.

Since $AC \cong BC$, $AP \cong BP$ and $CP \cong CP$, we observe by SSS that triangles $\triangle APC \cong \triangle BPC$. Thus, all the corresponding angles of $\triangle APC$ and $\triangle BPC$ are congruent, and in particular, $\angle CPA \cong \angle CPB$.

If we create a line m from C through P, we observe that the angles $\angle CPA$ and $\angle CPB$ formed at the intersection of lines l and m are congruent, and therefore, m is perpendicular to l. Since m runs through P, the construction is complete. □

32

Problem 11: The equivalence of SSS Triangle Congruency and Angle Transport

Lawrence Teng

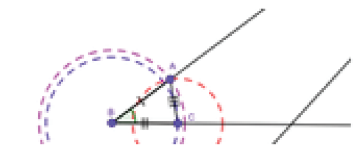
Axiom IV (SSS Triangle Congruency) If, given two triangles, $\triangle ABC$ and $\triangle DEF$, the following congruencies hold, $AB \cong DE$, $BC \cong EF$, and $AC \cong DF$, then $\triangle ABC \cong \triangle DEF$.

Axiom IV' (Angle Transport) Given an angle $\angle ABC$ and a ray $\overrightarrow{PQ}$ in the plane, you can construct an angle $\angle QPR$ congruent to $\angle ABC$.

Theorem 1.11. Axiom IV (SSS Triangle Congruency) is equivalent to Axiom IV' (Angle Transport).

Proof. We will first prove that Axiom IV implies Axiom IV'. Consider an angle $\angle ABC$ and a ray $\overrightarrow{PQ}$ in the plane. Construct the segment AC. We will now construct a triangle congruent to $\triangle ABC$ with a side on $\overrightarrow{PQ}$. Using Axiom III, construct a circle centered at P with radius BC, and label the intersection point between this circle and $\overrightarrow{PQ}$ as D. Again using Axiom III, construct a circle centered at P with radius BA, and construct a circle centered at D with radius CA. Label the intersection point between these last two circles as R. By construction, $PD \cong BC$, $PR \cong BA$, and $DR \cong CA$. Thus, by Axiom IV, $\triangle ABC \cong \triangle RPD$.

By triangle congruency, $\angle ABC \cong \angle RPD$. Since P, D, and Q are all collinear, $\angle RPD \cong \angle RPQ$. Thus, by transitivity, $\angle ABC \cong \angle RPQ$. We have accomplished the intended construction and shown that Axiom IV implies Axiom IV'.



Math 431: Euclidean Geometry

"This courses give students the tools to tackle an unpredictable array of geometric topics they might encounter in their future teaching practice. The aims of the course include that (1) students become comfortable operating within an axiomatic development of Euclidean Geometry; (2) students become critical, proficient provers and problem-solvers; and (3) students improve their mathematical communication."

Class Journal Description

The Math 431 class journal, *Communications in Euclidean Geometry,* will be soliciting articles from you this semester. In the same fashion as professional mathematicians, upon submission, your work will be reviewed by a referee and you will be able to address their feedback. To be published, entries must be clear, correct, complete, and well-written.

Grade Guildelines (30% of course grade):

- Earning an A means haveing 6–8 published articles, at least two should be notably deep or challenging or creative.
- Earning a B means 4-5 published articles.
- Earning a C means 2-3 publichsed articles.

Assignment Features/Goals

- Students can explore problems that most interest them
- Students can tackle problems too hard for general homework
- All students get ``solutions guide" to all homework problems
- Reveiw process mirrors real journal process
- Students have a polished artifact of class accomplishments
- Students can see how other students write about mathematics
- Students can see multiple, in-depth solutions to the same problem
- Students take feedback into account much more readily than other homework
- Students learn to typeset mathematics in LaTeX
- High standard of mathematical writing and communication

Challenges and Changes

Challenging workflow in review process. **Solution?** Create gmail address for "editor" and use labels religiously.

Student buy-in. **Change?** Sell idea early and often, get them to explain benefits themslves, make expectations reasonable.

Sign-up process. **Solution?** Make sure certain students don't sign up for too many articles. Encourage multiple, distinct solutions.